

# Image Analysis using LU, SVD and Kronecker Algebra in Multimodal Biometric Authentication System

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**Abstract-** Accurate uniqueness validation is critical in several aspects of our life. In early days prior to computing era, security is ensured by verifying the person personally and also with the help of the signature. Traditional authentication is inefficient as anyone can imprint as a real person with medical procedure and through spoofing. Now a day's authentication is done by online or offline by capturing the unique features such as biometrics of a person. The advantage of these unique in nature so that no one can represent the real person. To process these biometric qualities to achieve authentication is very complex. To improve the exactness, researches proposed different algorithms.

In this paper, by considering face and finger of a person and by using decomposing and reconstruction techniques such as Lower and Upper (LU) factorization, Singular Value Decomposition (SVD), and applications of Kronecker product we implemented multimodal authentication system. This system gives better result when compared to others.

**Index Terms-** Biometric, LU factorization, SVD, Katri Rao product and Kronecker product.

## 1. INTRODUCTION

Our society is electronically connected with the rapid growth of Information technology. With the vast usage of inter-connected electronic devices the daily transactions happened between individuals and organization such as banking and so on is increasing at stupendous rate. Biometric identification is gaining more importance day by day. Now, biometric indicators such as hand geometry, hand vein, face, facial thermo gram, fingerprint, ear, iris, signature, and voice print are widely used by the researchers for intensive evaluation [1]. Different mathematical models are available and each of them having its own advantages and disadvantages related to the acceptance and performance [2].

Authentication systems using biometrics can be categorized in two ways: One is unimodal biometric system – in this single biometric trait can be used for authentication and another is multimodal authentication system by using combination of two or more biometric traits. In order to operate biometric

system efficiently for authentication in diverse environments, a multimodal biometric system [3] is preferred.

The proposed multimodal biometric authentication system works by pre-processing the image using Principle Component Analysis (PCA) and then pre-processed image patterns will be represented as vector using vector algebra for the feature extraction. Feature extraction is completed using Lower and Upper (LU) factorization and Singular Value Decomposition (SVD). Finally, we applied convolution technique Katri Rao product which is a variant of Kronecker product for evaluating the features. An overview of the computational framework for the proposed model is described in figure 1.

## 2. FEATURE SELECTION AND EXTRACTION

### 2.1. Principal Component Analysis

Principal component analysis is a algebraic method for interpretation in multiple channels through an

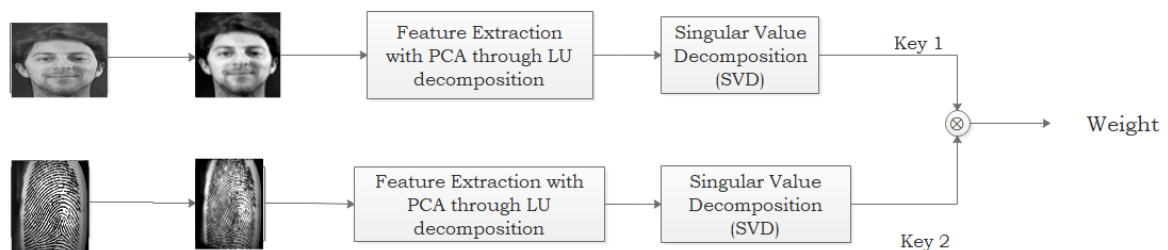


Fig. 1 Framework of the proposed model

orthonormal projection. Not only does principal component analysis have the ability to remove

uncorrelated noise, but also to decompose an image into constituent components. PCA has been in different applications including data transmission, compression and digital image enhancement [6]. PCA destructs a multi channel signal or an image into a set of orthogonal bases of decreasing energy. These sets can be reconstructed into the original signal exactly, or a truncated set can be used in reconstruction that tends to eliminate noise. If the matrix of eigen vectors sorted according to eigen value by  $\tilde{U}$ , then PCA transformation of the data as  $Y = \tilde{U}^T X$ . The eigen vectors are called principle components [5].

**2.2. LU Factorization**

Lower and Upper (LU) decomposition of a matrix representation is a mathematical method in the field of Image Processing and signal processing[4]. It is the multiplication of lower and upper triangle matrices. Sometimes it can also be considered as Gaussian Elimination.

Let A be a NxN matrix, it can be written as a product of lower( L) and upper (U) matrices as  $LU=A$

ie

$$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & 0 \\ u_{31} & 0 & 0 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

it can easily verify that  $a_{11} = m_{11}n_{11}$ . If  $k_{11} = 0$ , either  $m_{11} = 0$  or  $n_{11} = 0$  which shows that either matrix m=0 or matrix n=0. The matrix m is equivalent to L and n is equivalent to U.

**2.3. Singular value Decomposition (SVD)**

A matrix A of size mxn can be decomposed as [10]

$$A = UDV^T$$

U is a column orthogonal matrix of size mxn and its columns are eigen vectors of  $AA^T$

i.e,  $AA^T = UDV^TVDU^T = UD^2U^T$

V is a orthogonal matrix of size nxn and its columns are eigen vectors of  $A^T A$

i.e,  $A^T A = VDU^TUDV^T = VD^2V^T$

D is a diagonal matrix of size nxn called singular values.

If  $U = (u_1 u_2 \dots u_n)$  and  $V = (v_1 v_2 \dots v_n)$  then

$$A = \sum_{i=1}^n \sigma_i u_i v_i^T$$

**2.4. Kronecker Product**

The symbol  $\otimes$  can be used to represent the kronecker product. Let X and Y are two matrices then the kronecker product of two matrices can be represented as  $X \otimes Y$ . Let the matrix X of size p x q and the matrix Y of size r x s then the resulting matrix of size pr x qs

$$X \otimes Y = \begin{bmatrix} x_{11}Y & \dots & x_{1q}Y \\ \vdots & & \vdots \\ x_{p1}Y & \dots & x_{pq}Y \end{bmatrix}$$

**2.5 Katric Rao Product**

The Khatri-Rao product is a column-wise Kronecker product. Originally introduced by Khatri and Rao (1968) [7].

Given matrices  $A \in R^{I \times K}$  and  $B \in R^{J \times K}$  their Khatri-Rao product is denoted by  $A \otimes B$ . The result is a matrix of size  $(IJ) \times K(IJ) \times K$  and defined by [8]

$$A \otimes B = [a_1 \otimes b_1 \ a_2 \otimes b_2 \ \dots \ a_k \otimes b_k]$$

**3. DECISION STRATEGY – MEAN SQUARE ERROR (MSE)**

In the proposed model, the MSE is used as for verification with the support of feature and selection process at the training and testing levels.

**Mean Square Error:**

The Mean Square Error (MSE) of an estimator  $\hat{X}$  of a parameter X is the function of X

defined by  $E(\hat{X} - X)^2$  and it is denoted as  $MSE_{\hat{X}}$ .

$$MSE(\hat{X}) = E[(\hat{X} - X)^2]$$

The Mean Square Error is equal to the sum of the variance and the squared bias of the estimator or of the predictions. In the case of the MSE of an estimator,

$$MSE(\hat{X}) = Var(\hat{X}) + (Bias(\hat{X}, X))^2$$

**4. EXPERIMENT ANALYSIS**

By considering benchmark Yale and AT&T, FERET data sets [12,13], several experiments have been conducted. The MSE is considered as decision process. The experimental results on the chosen data is listed in the Table 1 with different key sizes such as 8x8, 16 x16,24x24,32x32 .....64x64 on similar and dissimilar patterns.

This model uses PCA for normalization and LU factorization for feature selection and extraction [5]. Later trained images can be projected by using SVD and weights can be calculated. These weights can be encoded with Katri Rao product [7]. Similarly testing image weights can be calculated and these weights can be compared with training image weights. Mean Square Error (MSE) was used for comparison. In this, we took, threshold value  $d=0.12$  and the results obtained were implemented on Core i7 7th generation processor with 16 GB of RAM. From the observations of MSE and False Acceptance Rate (FAR) and False Non Acceptance Rate (FNAR) have been measured for the chosen key sizes of the patterns. The algorithm works better for all key sizes.

S No	Key Size	Mean Square Error (MSE)		
		Completely Similar	Similar with different poses	Dissimilar
1	8x8	0.000000	0.040750	0.182083
2	16x16	0.000000	0.106621	0.136280
3	24x24	0.000000	0.108528	0.141979
4	32x32	0.000000	0.093009	0.165426
5	40x40	0.000000	0.114700	0.152512
6	48x48	0.000000	0.086422	0.195976
7	56x56	0.000000	0.076474	0.187708
8	64x64	0.000000	0.091097	0.205079

Table 1: Results

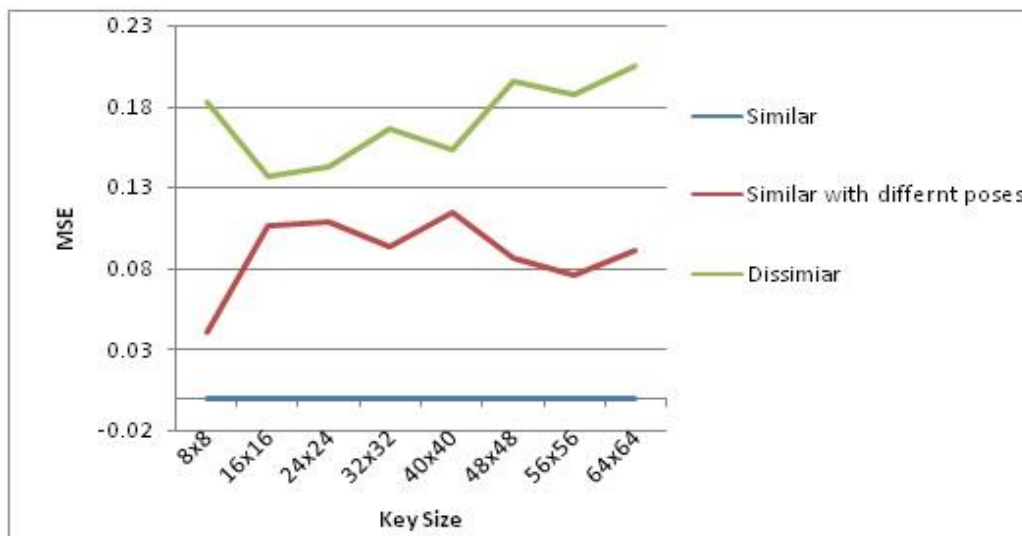


Fig.2. Graphical representation of results.

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